

What can we learn from neutrinoless double beta decay experiments?

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We assess how well next-generation neutrinoless double beta decay and normal neutrino beta decay experiments can answer four fundamental questions. (1) If neutrinoless double beta decay searches do not detect a signal, and if the spectrum is known to be inverted hierarchy, can we conclude that neutrinos are Dirac particles? (2) If neutrinoless double beta decay searches are negative and a next-generation ordinary beta decay experiment detects the neutrino mass scale, can we conclude that neutrinos are Dirac particles? (3) If neutrinoless double beta decay is observed with a large neutrino mass element, what is the total mass in neutrinos? (4) If neutrinoless double beta decay is observed, but next-generation beta decay searches for a neutrino mass only set a mass upper limit, can we establish whether the mass hierarchy is normal or inverted? We base our answers on the expected performance of next-generation neutrinoless double beta decay experiments and on simulations of the accuracy of calculations of nuclear matrix elements.

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I. INTRODUCTION

A new generation of double beta decay experiments will be undertaken with unprecedented accuracy. In approximately the same time frame, it will become possible to make much more precise measurements of, or set constraints on, the mass of neutrinos emitted in ordinary beta decay. The results of these next-generation experiments will be important for understanding the physics of weak interactions.

If neutrinoless double beta decay is observed, then one can conclude [1] immediately that neutrinos are Majorana particles without messing around with detailed calculations and qualifications of the kind discussed in this paper. (We will not consider alternative interpretations, such as *R*-parity violation [2–4], which can probably be verified or excluded at high-energy colliders. The violation of the lepton number is clear in either case.) The community of physicists can and will celebrate if double beta decay is observed.

In this paper, we provide quantitative estimates of how well we can answer four other fundamental questions about neutrinos using the assumed results of the next generation of neutrinoless double beta decay experiments and normal beta decay experiments.

Our principal results are summarized in Table I.

A. How can we estimate the uncertainties in calculated nuclear matrix elements?

The uncertainty in the calculated nuclear matrix elements for neutrinoless double beta decay will constitute the principal obstacle to answering some basic questions about neutrinos. The essential problem is that the correct theory of nuclei is QCD, a notoriously difficult theory with which to do cal-

culations for nuclei with several nucleons. For neutrinoless double beta decay, the situation is even more severe because double beta candidates involve systems with $A \sim 50$ to $A \sim 100$ and even larger. Very attractive next-generation experiments have been proposed for a number of different isotopes, including ^{48}Ca [6], ^{76}Ge [5,7,8], ^{100}Mo [9,10], ^{116}Cd [11], ^{130}Te [12,13], ^{136}Xe [14–16], ^{150}Nd [17], and ^{160}Gd [18,19].

In the foreseeable future, it does not seem possible to derive in a direct and controlled manner from QCD nuclear matrix elements for large A . Thus there is no way of quantifying with absolute confidence the range of uncertainties in nuclear matrix elements calculated with different theoretical models or approximations.

In the absence of being able to derive the errors directly from QCD, we assume that the published range of calculated matrix elements defines a plausible approximation to the uncertainty in our knowledge of the matrix elements. We do not, for example, favor a particular calculation because it happens to give better agreement with the inferred matrix element for two-neutrino double beta decay (in the rare cases where this decay has been observed). We have no way of knowing for sure what the improved agreement for the two-neutrino case implies for the neutrinoless double beta decay matrix element and whether, indeed, the agreement in a special case is accidental or not.¹

We recognize that different individuals may regard the calculated range of nuclear matrix elements as either too narrow or too broad to reflect the actual uncertainty. However, we do not know of any way to settle objectively and conclusively whether our estimate of the uncertainty is pessimistic or optimistic in any particular case.

¹Fukugita and Yanagita [20] note that the nuclear levels that are important for neutrinoless double beta decay are typically at excitation energies of order 10 MeV, while for two neutrino double beta decay the characteristic excitation energies are lower, a few MeV. Thus even if the lower excitation states are correctly described, there is no guarantee that the higher excitation states are also correctly described.

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TABLE I. Answers to some questions about the potential of neutrinoless double beta decay experiments. Answers refer to a C.L. of 99.73 % C.L. for the assumed probability distributions. We adopt a sensitivity s equal to what is projected for the Majorana experiment [5] (if another reference sensitivity s' is assumed, the required number of experiments should be scaled by $N'_{exp} = N_{exp}s/s'$). If the answer for an inverted neutrino mass hierarchy is different from the answer for a normal mass hierarchy (see Fig. 2), we show in parentheses the answer for a normal mass hierarchy.

Section	Assumptions	Question	N_{exp} at 99.73 % C.L.
II	No detected neutrinoless double β decay	Dirac ?	230 (∞)
III	lightest mass scale (1 ± 0.05 eV), No neutrinoless double β decay	Dirac ?	1
III	lightest mass scale (0.35 ± 0.07 eV), No neutrinoless double β decay	Dirac ?	5 (6)
III	lightest mass scale (0.3 ± 0.1 eV), No neutrinoless double β decay	Dirac ?	16 (∞)
IV	Neutrinoless double β decay: $T_{1/2}({}^{76}\text{Ge}) = (3.2 \pm 0.2) \times 10^{25}$ yr	Total mass ?	[0.46,9.56] ([0.48,9.58])
IV	Neutrinoless double β decay: $T_{1/2}({}^{76}\text{Ge}) = (1. \pm 0.1) \times 10^{26}$ yr	Total mass ?	[0.24,8.34] ([0.28,8.40])
IV	Neutrinoless double β decay: $T_{1/2}({}^{76}\text{Ge}) = (3.2 \pm 0.5) \times 10^{26}$ yr	Total mass ?	[0.08,5.68] ([0.16,6.06])
V	Detected neutrinoless double β decay	Hierarchy ?	No
V	Detected neutrinoless double β decay, private communication: $m=0$	Hierarchy ?	Yes

B. Some definitions

The neutrino mass matrix element that appears in neutrinoless double beta decay [21–23] is given by

$$|\langle m_{ee}^{\nu} \rangle| = m_e \frac{1}{\sqrt{T_{1/2} F_N}} = m_e \sqrt{\frac{\lambda}{\ln 2 F_N}}, \quad (1)$$

where m_e is the electron mass, $T_{1/2}(\lambda)$ is the half life (exponential decay constant) of the double beta decay process, and the nuclear structure parameter F_N is given by

$$F_N = G^{0\nu} \left| M_F^{0\nu} - \left(\frac{g_A}{g_V} \right)^2 M_{GT}^{0\nu} \right|^2. \quad (2)$$

For specificity, we consider a neutrinoless double beta decay experiment with sensitivity to $T_{1/2} F_N$ that is exemplified by what is expected for the Majorana experiment [5] (see also, compilation in Ref. [22]). We will consider that a number N_{exp} of neutrinoless double beta decay experiments are performed with the expected Majorana sensitivity s .

Our results are, however, general. If the experiments that actually are or could be performed have a different sensitivity, then our results should be rescaled by

$$N'_{exp} = N_{exp} s/s'. \quad (3)$$

If a specific neutrinoless double beta decay experiment successfully detects a signal, then a greatly increased exposure with the same detector will not improve much the confidence

with which one can answer the questions raised in this paper. For a single detector, the uncertainty will be dominated by the nuclear factor of that nucleus. Measurements with different nuclei will be required to improve the statistical significance of the answers to questions about the nature and properties of neutrinos. On the other hand, suppose the search for neutrinoless double beta decay is negative with a given detector. Then an increase in the exposure time by a factor $N_{exposure}$ is equivalent to performing $N_{exposure}$ new experiments that have the identical sensitivity.

The neutrino mass element $|\langle m_{ee}^{\nu} \rangle|$ is related to the fundamental neutrino parameters by the expression

$$|\langle m_{ee}^{\nu} \rangle| = |m_1| U_{e1}^2 |e^{i\phi_1} + m_2| U_{e2}^2 |e^{i\phi_2} + m_3| U_{e3}^2|, \quad (4)$$

where m_i are the mass eigenvalues of the Majorana neutrinos, U is the lepton mixing matrix, and ϕ_i are relative Majorana phases. Normal (inverted) hierarchy corresponds to the ratio between mass eigenvalues (labeled in increasing mass eigenvalue $m_1 < m_2 < m_3$) given by $m_3/m_2 > m_2/m_1$ ($m_3/m_2 < m_2/m_1$). If hierarchies are indistinguishable, what happens when $\Delta m_{ij}^2 \ll m_1^2$? Then, the mass scheme is called degenerate.

C. The dispersion in calculated nuclear matrix elements

The dispersion of the calculated nuclear matrix elements obtained by different theoretical methods is large. For example, a compilation of 20 different calculations [5,24,25] for ${}^{76}\text{Ge}$ spans the range $2.7 \times 10^{-15} - 2.9 \times 10^{-13} \text{ yr}^{-1}$.

Figure 1 shows the distribution of ${}^{76}\text{Ge}$ nuclear factors binned in a logarithmic scale. In our analyses of how much

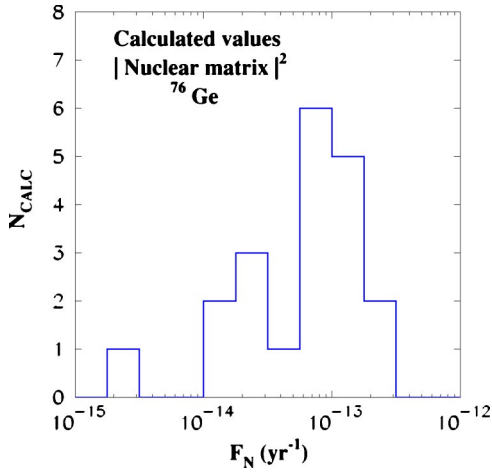


FIG. 1. Distribution of ^{76}Ge nuclear factor results. A compilation of 20 different calculations spans in the range $2.7 \times 10^{-15} - 2.9 \times 10^{-13} \text{ yr}^{-1}$ [5].

we can learn about different fundamental neutrino questions, we will also consider F_N as a random variable in linear and logarithmic scales of the constant and the Gaussian probability distributions. For the Gaussian distribution, we will adopt the central value of the F_N interval as the mean and one third of the radius of the interval covered by calculated values of F_N as the standard deviation. The lowest nuclear factor F_N shown in Fig. 1 corresponds to a recent calculation [26] that used a self-consistent renormalized quasiparticle random phase approximation. We do not know of any rigorous argument that would exclude this recent calculation while including the other calculations shown in the figure.

For the numerical calculations given in this paper, we used the distribution of calculated nuclear factor for ^{76}Ge because this nucleus is the one for which we found the largest number of published calculations of F_N . We performed Kolmogorov-Smirnov tests to test if the distributions of F_N that were calculated for other double beta decay candidates (^{82}Se , ^{130}Te , ^{136}Xe) are consistent with the distribution shown in Fig. 1 of ^{76}Ge . Table 2 of Ref. [22] compiles a list of six calculations [27] for these nuclei. The Kolmogorov-Smirnov tests show that we cannot reject at 95 % C.L., for any of the nuclei ^{82}Se , ^{130}Te , or ^{136}Xe , the hypothesis that the distribution of calculations of F_N given in Table 2 of Ref. [22] is the same distribution as shown in Fig. 1 for ^{76}Ge . We also checked that the distribution of the six calculations listed in Table 2 of Ref. [22] for ^{76}Ge is consistent with the distribution of 20 calculations of F_N used in the present work.

The fact that the uncertainty in the nuclear matrix element plays a major role in our ability to resolve fundamental questions in neutrinoless double beta decay experiments is well known (see for example the famous reviews in Ref. [23]). Reference [28] is the most recent example with which we are familiar of a systematic analysis that assumes a small uncertainty in the nuclear matrix elements for neutrinoless double beta decay experiments (for the nuclear physics discussion see Ref. [29]). The discussion in Ref. [28] assumes the correctness of the renormalized quasiparticle random phase ap-

proximation (RQRPA) that leads to the lowest nuclear factor in Fig. 1. Readers who are optimistic regarding the validity of current calculational methods for calculating nuclear matrix elements in neutrinoless double beta decay may prefer the conclusions of Ref. [28] instead of the more conservative conclusions of the present paper.

The position adopted in this paper is that the RQRPA could be accurate, or some other calculational scheme could be more accurate, but we will not know for sure how precise any approximation is until calculations can be done in a controlled manner using QCD. Our attitude is consistent with the point of view expressed in the recent discussion of the RQRPA and QRPA approximations in Ref. [29]. These authors summarized their analysis with the statement [29]: “Even though we cannot guarantee this basic method [RQRPA] is trustworthy, we have eliminated, or at least greatly reduced, the arbitrariness commonly present in published calculations.” In other words, the recommended prescription results in a small dispersion in calculated nuclear matrix elements, which may or may not be close to the true value.

The reader will choose what to believe based upon the reader’s convictions about the accuracy of the calculations of nuclear matrix elements. We believe that the burden of proof is on the person drawing conclusions that depend on the size of the nuclear matrix elements. The conclusions must be supported by a proof that the matrix elements are equal to the QCD values within the stated errors.

Our goal is to provide, for the reader’s consideration, an alternative viewpoint to the one that is usually adopted in discussing neutrinoless double beta decay experiments. As far as we know, there is no previous systematic, quantitative study to evaluate the impact of the uncertainty in the nuclear matrix element for different assumed probability distributions. Recently, it has been demonstrated that it is not practical to detect in neutrinoless double beta decay experiments neutrino CP violation arising from Majorana phases [30,31].

D. How do we determine how many experiments are required?

For each question about neutrino properties that we address, we make specific assumptions about what is or is not observed experimentally. Depending on the particular question we are addressing, we will assume that the neutrino masses satisfy a normal or an inverted hierarchy, as illustrated in Fig. 2. We will also make assumptions regarding the observation, or nonobservation, of a neutrino mass in ordinary (tritium) beta-decay.

Figure 3 shows the relationship between the neutrinoless double beta decay mass element $\langle m_{ee}^{\nu} \rangle$ and the smallest neutrino mass m [32–34]. This figure plays a key role in our discussion; we will return to Fig. 3 in Secs. II, IV, and V.

For a given set of assumptions as described above, we compute the different probability distributions that are implied by the assumed experimental constraints. In the final step of our analysis, we combine the computed probability distributions in order to determine how many experiments

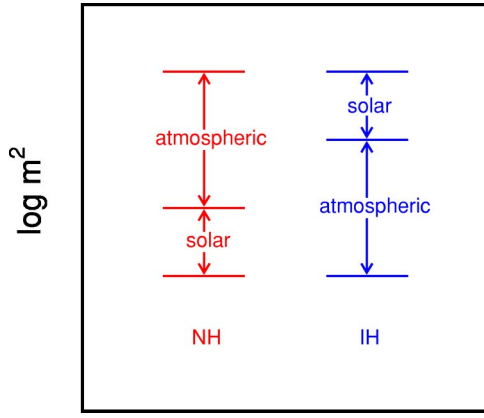


FIG. 2. Normal and inverted neutrino mass hierarchy. The larger splitting is $\Delta m_{\text{atm}}^2 \approx 2 \times 10^{-3} \text{ eV}^2$; the smaller splitting is $\Delta m_{\text{solar}}^2 \approx 7 \times 10^{-5} \text{ eV}^2$. The hierarchy is referred as “Degenerate” if the square of the smallest mass is much larger than either Δm_{atm}^2 or $\Delta m_{\text{solar}}^2$.

are required to answer a stated question at a specific confidence level.

E. What is the bottom line?

Some readers will only care about the bottom line. How many neutrinoless double beta decay experiments are required in order to determine whether neutrinos are Majorana or Dirac particles?² What fraction of the closure mass of the universe do neutrinos constitute? Can we establish whether the neutrino masses satisfy a normal or an inverted mass hierarchy?

Table I summarizes our numerical results. We state in column 2 of Table I the different assumptions that we have made about future experiments. In column 3, we give abbreviated names to the questions that we have asked. Finally, in column 4, we present a brief summary of our answers to the different physical questions about neutrinos. The reader interested in the details of how a specific question was answered can look in the section of this paper that is listed in column 1 of Table I.

F. Outline of this paper

In Sec. II, we show that an impractically large number of neutrinoless double beta decay experiments would be required to show that neutrinos are Dirac particles if next generation experiments do not reveal neutrinoless double beta decay. We show in Sec. III that nonobservation of neutrinoless double beta decay taken together with a measurement in ordinary beta decay of the lowest neutrino mass that is near the present upper limit (e.g., $\sim 1 \text{ eV}$) would be sufficient to

²Note that we are referring to the dominant neutrino masses relevant to the currently observed neutrino oscillation. Even if the dominant masses are Dirac, there may be much smaller Majorana masses not relevant to neutrino oscillation, sometimes called pseudo-Dirac. We do not distinguish Dirac and pseudo-Dirac neutrinos in this paper.

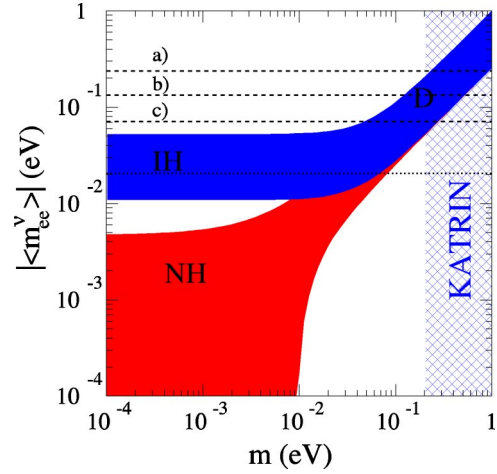


FIG. 3. The neutrinoless double beta decay mass element $|\langle m_{ee}^ν \rangle|$ versus the lowest neutrino mass m . The regions allowed at 90 % C.L. by existing neutrino oscillation data are shown for a normal neutrino hierarchy (NH), an inverted hierarchy (IH), and degenerate neutrinos (D) (see Fig. 2 for an explanation of the different hierarchical arrangements of neutrino masses and Sec. V for a description of how the allowed regions were computed). The hatched area shows the parameter space that can be excluded by the Katrin experiment [35] if no evidence for a neutrino mass is detected in tritium beta decay. The three dashed lines labeled (a), (b), and (c) refer to three possible positive results for a next generation neutrinoless double beta decay search and are discussed in Secs. IV and V. The dotted horizontal line near 10^{-2} eV illustrates the sensitivity that is expected for the Majorana experiment [5]. For an original version of this figure, see Ref. [32].

show that neutrinos are Dirac particles. However, if the neutrino mass is as low as 0.3 eV or lower, then many neutrinoless double beta decay experiments would be required to show that neutrinos are Dirac particles. We present in Sec. IV the allowed ranges in the total mass in neutrinos if neutrinoless double beta decay is detected at different possible half-lives. Finally, we show in Sec. V that even if neutrinoless double beta decay is observed in next generation experiments we nevertheless will not be able to decide from beta decay experiments alone whether the mass hierarchy is normal or inverted. We discuss our principal results in Sec. VI.

II. ARE NEUTRINOS DIRAC PARTICLES? NO NEUTRINOLESS DOUBLE BETA DECAY AND INVERTED HIERARCHY

In this section, we assume that next generation experiments [16] will not observe neutrinoless double beta decay. Figure 3 shows that it is much easier to observe neutrinoless double beta decay if the neutrino mass hierarchy is inverted. If the hierarchy is normal, then the neutrino mass matrix element, $|\langle m_{ee}^ν \rangle|$, can be unobservably small even if neutrinos are Majorana particles, making it impossible to decide for a normal hierarchy whether neutrinos are Dirac or Majorana. Hence, we concentrate our numerical calculations in this section on the case in which the mass hierarchy is known to be inverted from long baseline experiments [36,37] or from some other measurement.

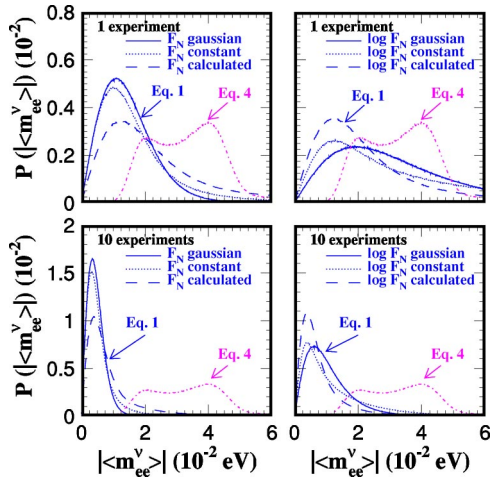


FIG. 4. Probability distributions of $|\langle m_{ee}^\nu \rangle|$ in future generation neutrinoless double beta decay experiments. On the right side of each of the four panels, we plot the pdf of $|\langle m_{ee}^\nu \rangle|$, which is obtained from neutrino oscillation data by using Eq. (5) (dashed-dotted line). On the left side of each of the four panels of the figure, we show probability distribution functions for $|\langle m_{ee}^\nu \rangle|$ assuming that next generation experiments do not detect neutrinoless double beta decay. The plotted pdfs were obtained by making different assumptions regarding the pdf of the nuclear factor F_N that appears in Eq. (1). For the left-hand-side panels, we assume that F_N follows a Gaussian distribution (full line), a constant distribution (dotted line), or the actual computed distribution of values of F_N computed by different nuclear theorists (dashed line). For the right-hand-side panels, we assumed that $\log F_N$ follows these same distributions. The upper pair of panels corresponds to a single experiment with a sensitivity equal to what is expected for the Majorana experiment [5]. The lower panels correspond to simulated results for ten experiments each with a sensitivity equal to the anticipated sensitivity of the Majorana experiment [5].

For definiteness and in order to minimize the number of required experiments, we assume that the data are free of all background and that there are no candidate neutrinoless double beta decay events. The decay constant λ then satisfies an exponential probability decay function (pdf) corresponding to the Poisson probability that no events are observed.

Given that we know that there is an inverted mass hierarchy for neutrinos, how many neutrinoless double beta decay experiments would be needed to establish that neutrinos are Dirac particles at a given C.L.? We shall see that in this case 230 neutrinoless double beta-decay experiments are required in order to establish that neutrinos are Dirac particles at a C.L. equivalent to 3σ . If we admitted that there is a possibility that the neutrino hierarchy is normal, then an essentially infinite number of experiments would be required.

In order to calculate the required number of experiments, we first compute the probability distribution function of the neutrino mass element $|\langle m_{ee}^\nu \rangle|$ given by Eq. (1). This pdf depends upon the assumed distribution of the nuclear matrix element F_N .

Figure 4 illustrates the dependence of the probability distribution function of $|\langle m_{ee}^\nu \rangle|$ on the pdf of the nuclear factor F_N . We show the calculated pdfs for $|\langle m_{ee}^\nu \rangle|$ that follow

TABLE II. No neutrinoless double beta decay plus inverted hierarchy. The table gives the number of neutrinoless double beta decay experiments with sensitivity to $|\langle m_{ee}^\nu \rangle|$ equal to what is projected for the Majorana [5] experiment that are required to show that neutrinos are not Majorana particles at 90, 95, 99, and 99.73 % C.L. if an inverted hierarchy is correct (see Fig. 2). We consider different probability distributions of the nuclear factor, F_N : Gaussian, constant, or the actual distribution of 20 different calculations (see Fig. 1); either using linear (lin) or logarithmic (log) scales.

F_N pdf	N_{exp} at 90 % C.L.	N_{exp} at 95 % C.L.	N_{exp} at 99 % C.L.	N_{exp} at 99.73 % C.L.
actual, lin	11	21	81	230
actual, log	9	17	61	141
Gaussian, lin	3	4	8	13
Gaussian, log	16	23	50	83
constant, lin	4	7	21	45
constant, log	24	40	95	156

from Eq. (1) for different assumptions about the pdf of F_N : a Gaussian (full line), a constant probability spanning the entire range of calculated F_N (dashed line), and a pdf equal to the actual reported distribution of F_N (dashed line). In the computations shown in the upper and lower left-hand corners of Fig. 4, the value of F_N is treated as the random variable with the illustrated pdf, while in the upper and lower right-hand corners, the pdfs were calculated treating the logarithm of F_N as the random variable. The two lower panels of Fig. 4 are similar to the two upper panels except for the fact that the lower panels refer to the pdfs computed assuming ten equivalent experiments (equal sensitivity) have been performed instead of just one experiment.

We next concentrate on the neutrino mass element $|\langle m_{ee}^\nu \rangle|$ as a function of the neutrino parameters. In the case of inverted hierarchy, the appropriate expression for $|\langle m_{ee}^\nu \rangle|$ is given by

$$\begin{aligned}
 |\langle m_{ee}^\nu \rangle|_{IH} = & m \sin^2 \theta_{13} \\
 & + \cos^2 \theta_{13} (\cos^2 \theta_\odot \sqrt{m^2 + \Delta m_{atm}^2} - \Delta m_\odot^2 e^{i\phi_1} \\
 & + \sin^2 \theta_\odot \sqrt{m^2 + \Delta m_{atm}^2} e^{i\phi_2}), \quad (5)
 \end{aligned}$$

where m is the mass of the lowest mass eigenstate; $\Delta m_\odot^2$ and Δm_{atm}^2 are mass-squared splittings; and $\theta_\odot$ and θ_{13} are mixing angles determined by solar, atmospheric, reactor, and K2K experiments [38]. We have computed numerically the pdf of the neutrino mass element that corresponds to Eq. (5). In this computation, we used Gaussian distributions for the mass-squared splittings and mixing angles, with mean values and standard deviations given by $\Delta m_\odot^2 = (7.1 \pm 0.7) \times 10^{-5} \text{ eV}^2$, $\Delta m_{atm}^2 = (2.0 \pm 0.4) \times 10^{-3} \text{ eV}^2$, $\sin^2 \theta_\odot = 0.30 \pm 0.03$, and $\sin^2 \theta_{13} = 0.008 \pm 0.02$ [38–40]. In the latter case, we truncate the Gaussian distribution to include only positive values. We assumed constant probability distributions for the lightest mass m (in logarithmic scale, with $10^{-6} < m < 2.3 \text{ eV}$) and the phases ϕ_1 and ϕ_2 (in linear scale).

In order to compute the number of experiments required to establish that neutrinos are Dirac particles, we must compute the joint probability P that follows from the unobserved neutrinoless double beta decay experiments [see Eq. (1)] and from the expression for the neutrino mass

$|\langle m_{ee}^\nu \rangle|$ in terms of various neutrino oscillation parameters [see Eq. (4)]. The probability P that these two conditions are satisfied is given by the product of the probabilities of the two individual constraints, conveniently normalized. Thus

$$P(N_{exp}) = \frac{\int d|\langle m_{ee}^\nu \rangle| \int d|\langle m_{ee}^\nu \rangle'| P_1(|\langle m_{ee}^\nu \rangle|, N_{exp}) P_2(|\langle m_{ee}^\nu \rangle'|) \delta(|\langle m_{ee}^\nu \rangle| - |\langle m_{ee}^\nu \rangle'|)}{\sqrt{\int P_1^2(|\langle m_{ee}^\nu \rangle|, N_{exp}) d|\langle m_{ee}^\nu \rangle|} \sqrt{\int P_2^2(|\langle m_{ee}^\nu \rangle'|) d|\langle m_{ee}^\nu \rangle'|}} \quad (6)$$

In Table II we show the number of experiments needed to reject, at 90, 95, 99, and 99.73 % C.L. (3σ), the hypothesis that neutrinos are Majorana particles. In all cases, we find that many experiments are required in order to establish that neutrinos are not Majorana particles. However, there is a wide dispersion, from 13 experiments to 250 experiments at 3σ , in the number of experiments that is required depending on the assumed pdf of the nuclear factor F_N .³

For the bottom-line table, Table I, we adopt the most conservative case in which the assumed probability distribution function for F_N is given by the actual calculated distribution of F_N values. In any event, Table I shows that a few next-generation neutrinoless double beta decay experiments will not be able to answer the question of whether neutrinos are Dirac or Majorana particles unless neutrinoless double beta decay is actually observed.⁴

What could be the effect of future improvement in the neutrino oscillation parameters, such as $\sin^2\theta_{12}$ from a SNO study with neutral current detectors (NCD) [43], and Δm_{23}^2 from NuMI/MINOS [44] and T2K experiments [45]? In curves labeled “Eq. 4” in Fig. 4, the two peaks correspond to the maximally constructive (right) and destructive (left) case

in Eq. (5), connected by a plateau due to the randomly assigned complex phases. On the other hand, the tails above and below the peaks are mostly due to the uncertainties in $\sin^2\theta_{12}$ and Δm_{23}^2 . It is clear that the improvements in measurements cannot change the situation qualitatively. We have performed the same analysis with twice as accurate measurements or with no errors at all, and found that the numbers in Table II cannot change more than 40 %. For example, if we assume that all of the neutrino oscillation parameters are known with infinite precision, the required number of experiments to obtain a 3σ result is reduced from 230 to 156.

III. ARE NEUTRINOS DIRAC PARTICLES? NO NEUTRINOLESS DOUBLE BETA DECAY, BUT NEUTRINO MASS MEASURED

In this section, we make two assumptions.

- (i) Next generation experiments [16] do not observe neutrinoless double beta decay.
- (ii) Next generation beta decay experiments [35] observe the neutrino mass scale.

The first assumption is identical to our first assumption in Sec. II. The second assumption assumes that an experiment with the expected sensitivity of the KATRIN experiment [35] will successfully identify a spectral distortion of the tritium beta decay energy spectrum that is due to a finite neutrino mass.

Given the measurement of a neutrino mass in the KATRIN experiment, how many double beta experiments would we need to establish that neutrinos are Dirac particles at a given C.L.?

We will consider three cases. First, $m = 1$ eV, which is chosen because this value is close to the present upper bound for a neutrino mass in ordinary beta decay [46,47]. Second, $m = 0.35$ eV, which is chosen because this is the smallest mass that could be discovered at 5σ in next-generation experiments that perform with the sensitivity of the KATRIN experiment [35]. Third, $m = 0.30$ eV, which is chosen because it is the smallest mass that could be discovered at 3σ in a next-generation experiment with the expected KATRIN sensitivity [35]. We assume that m is normally distributed

³We checked our results by comparing with a conservative case. We assumed that the lightest neutrino mass is zero, neglected θ_{13} and the solar mass splitting, and chose the Majorana phase to be π . In this special case, the neutrinoless double beta mass element has a lower limit [32–34,39,41,42]. We can compute straightforwardly the probability that the mass matrix element derived from negative searches is higher than the lower bound at a given confidence level. As expected, this calculation indicates more experiments are required than we found are necessary using the full probability distributions. For example, in the case “actual, lin” the calculation using the lower bound gives 14 (550) required experiments at 90 % C.L. (3σ).

⁴We checked that our results do not depend very sensitively upon the assumption that equal decades in the lightest mass m are equally probable. We made instead the extreme assumption that the pdf of the mass m is equally distributed on a linear scale with $0 < m < 2.3$ eV. This optimistic assumption presumes that there is a 50 % chance that the lowest mass lies between 1.15 eV and 2.3 eV. Nevertheless, the required number of experiments at 3σ is 81.

with a mean value of 1.0 (0.35) [0.3] eV and standard deviation 0.05 (0.07) (0.10) eV for the three cases listed in the order given above.⁵

The last two cases, which are given as examples in the Majorana proposal [5], are separated by only 0.05 eV. However, as we shall see in the discussion below, this small difference in mass makes a large difference in the number of required experiments. The essential reason for this large difference is that if the experiment shows that $m = 0.35 \pm 0.07$ eV, then we know that m is well separated from zero mass at 3σ . However, if $m = 0.30 \pm 0.10$ eV, then at 3σ the lightest mass could be zero.

We are now in a position to compute the required number of neutrinoless double beta decay experiments. Compared to the analysis done in Sec. II, we need only modify our analysis of Eq. (5), replacing the pdf assumed in Sec. II for m by the corresponding Gaussian distribution that represents one of the three cases listed above for a next-generation beta decay experiment. Moreover, in this section we do calculations for both neutrino mass hierarchies, normal, and inverted [given by Eq. (5)]. For a normal hierarchy, the neutrino mass element can be written as

$$|\langle m_{ee}^{\nu} \rangle|_{NH} = |\cos^2 \theta_{13} (m \cos^2 \theta_{\odot} + \sqrt{m^2 + \Delta m_{\odot}^2} \sin^2 \theta_{\odot} e^{i\phi_1}) + \sqrt{m^2 + \Delta m_{atm}^2 + \Delta m_{\odot}^2} \sin^2 \theta_{13} e^{i\phi_2}|. \quad (7)$$

For the first case, $m = 1.0 \pm 0.05$ eV, one experiment is sufficient to prove that neutrinos are Dirac particles at more than 3σ ($P > 99.77\%$).

Table III presents for the second and third cases the results for different assumptions about the pdf of F_N . For the second case, $m = 0.35 \pm 0.07$ eV, one experiment is sufficient to prove that neutrinos are Dirac particles at 95 % C.L., but six experiments are required to prove that neutrinos are not Dirac particles at 3σ . For the third case, $m = 0.30 \pm 0.10$ eV, and assuming an inverted hierarchy, 2 experiments are sufficient to prove neutrinos are Dirac particles at 90 % C.L., but 16 experiments are required to prove that neutrinos are not Majorana particles at 3σ .

The differences between hierarchies are small in the first and second cases listed above because the mass scale m is assumed large compared with the solar and atmospheric mass splittings. In these two cases, the neutrino masses are essentially degenerate (imagine Fig. 2 for the case in which m is much larger than either the solar or the atmospheric mass splitting).

⁵It is possible that m^2 is normally distributed at the same significance rather than m . Then $m = 0.3 \pm 0.1$ eV is replaced by $m^2 = 0.09 \pm 0.03$ eV², and the effective error in m is reduced. In this case, we find that the number of experiments required to conclude neutrinos are Dirac particles is slightly larger [differ at most in 1(2) experiment(s) at 99 (99.73) % C.L.] than in the case $m = 0.35 \pm 0.07$ eV shown in Table III. On the other hand, it remains true that the sensitivity to the Majorana character of neutrinos quickly runs out of steam below 0.3 eV.

TABLE III. No neutrinoless double beta decay plus measured neutrino mass in ordinary beta decay. What do we need to know to conclude in this case that neutrinos are Dirac particles? The format of the table is similar to Table II except that for Table III we assume that a neutrino mass, m , has been detected in ordinary beta decay. The table gives results for two hypothesized cases, $m = 0.35 \pm 0.07$ eV [35] and $m = 0.30 \pm 0.10$ eV [35]. If the required number of experiments depends on whether the neutrino mass hierarchy is normal or inverted, then the result for the normal hierarchy is written in parentheses. The case $m = 1.0 \pm 0.05$ eV is discussed in Sec. III.

F_N pdf	N_{exp} at 90 % C.L.	N_{exp} at 95 % C.L.	N_{exp} at 99 % C.L.	N_{exp} at 99.73 % C.L.
$m = 0.35 \pm 0.07$ eV				
actual, lin	1	1	2	5(6)
actual, log	1	1	2	3(4)
Gaussian, lin	1	1	1	1
Gaussian, log	1	1	2	2
constant, lin	1	1	1	1
constant, log	1	1	2	4
$m = 0.3 \pm 0.1$ eV				
actual, lin	1	2	5(14)	15(∞)
actual, log	1	1(2)	4(10)	9(∞)
Gaussian, lin	1	1	1(3)	2(∞)
Gaussian, log	1	2	5(13)	10(∞)
constant, lin	1	1	2(4)	4(∞)
constant, log	1(2)	2(3)	7(19)	16(∞)

For the third case, $m = 0.030 \pm 0.10$ eV, there are large quantitative differences between the normal hierarchy and the inverted hierarchy at higher C.L. The reason for the difference in behavior can be seen visually in Fig. 3. The mass matrix element $|\langle m_{ee}^{\nu} \rangle|$ in a normal hierarchy can be greatly reduced because of cancellations, while m is bounded from below in an inverted mass hierarchy. For a normal hierarchy and small values of m , $|\langle m_{ee}^{\nu} \rangle|$ could be extremely small, orders of magnitude below the expected level of sensitivity of next-generation double beta decay experiments. Therefore, the corresponding entries in Table III require at 3σ C.L. an infinite number of next-generation double beta decay experiments to distinguish between Dirac and Majorana particles.

IV. WHAT IS THE TOTAL MASS IN NEUTRINOS? NEUTRINOLESS DOUBLE BETA DECAY DETECTED

We suppose in this section that next-generation experiments [16] successfully detect neutrinoless double beta with a large neutrino mass matrix element $|\langle m_{ee}^{\nu} \rangle|$.

We will compute in this section the pdf for the lowest mass eigenstate m using hypothesized results from next-generation neutrinoless double beta decay experiments. Since we already know from existing experiments the pdf for

the mass splittings $\Delta m_{\odot}^2$ and Δm_{atm}^2 , we can use these data together with the results for the lowest mass m to compute the cumulative pdf for the total mass in neutrinos.

We will use ^{76}Ge as an illustrative case. The Heidelberg-Moscow experiment [48] provides a lower limit on the half-life (we remind the reader that there is a claim of 4σ detection in Ref. [49]),

$$T_{1/2} > 1.9(3.1) \times 10^{25} \text{ yr} \quad (8)$$

at 90 % C.L. (68 % C.L.).

We consider three feasible cases with positive neutrinoless double beta detection: (a) $T_{1/2} = (3.2 \pm 0.2) \times 10^{25}$ yr, (b) $T_{1/2} = (1. \pm 0.1) \times 10^{26}$ yr, and (c) $T_{1/2} = (3.2 \pm 0.5) \times 10^{26}$ yr, corresponding to 373, 118, and 37.3 events expected in the parameter region of interest in the Majorana experiment [5]. The expected background in a deep underground experiment is 5.5 events, although background could be different by a factor of two. Systematic errors are expected to be a few percent and to be dominated by energy resolution, the segmentation cut, and the pulse-shape discrimination acceptance. Our results are not significantly affected by including systematic errors of a few percent because of the dominant contribution of the uncertainty in the nuclear factor F_N .

We computed numerically the pdf of the neutrino mass element $|\langle m_{ee}^\nu \rangle|$ given by Eq. (1) for all three values of $T_{1/2}$ listed above and for all six possible distributions of the nuclear factor F_N that were discussed in Secs. II and III. The determination of the neutrino mass element can be used to extract the probability distribution $P(m)$ of the lightest mass eigenstate by extracting with the help of Eqs. (5) and (7). In order to find $P(m)$, we compute

$$P(m) = \frac{\int d|\langle m_{ee}^\nu \rangle| \int d|\langle m_{ee}^\nu \rangle'| P_1(|\langle m_{ee}^\nu \rangle|) P_2(|\langle m_{ee}^\nu \rangle'|, m) \delta(|\langle m_{ee}^\nu \rangle| - |\langle m_{ee}^\nu \rangle'|)}{\sqrt{\int P_1^2(|\langle m_{ee}^\nu \rangle|) d|\langle m_{ee}^\nu \rangle|} \sqrt{\int P_2^2(|\langle m_{ee}^\nu \rangle'|, m) d|\langle m_{ee}^\nu \rangle'|}}, \quad (9)$$

where $P_1(|\langle m_{ee}^\nu \rangle|)$ is the pdf of the neutrino mass element given by Eq. (1) and $P_2(|\langle m_{ee}^\nu \rangle'|, m)$ is the pdf of the neutrino mass element given by Eq. (7) or (5), respectively, for a normal or an inverted neutrino mass hierarchy.

Figure 5 shows the computed pdfs of the lightest mass eigenstate in the case of normal and inverted neutrino mass hierarchies for different assumptions regarding the pdf of the nuclear factor F_N . For illustrative purposes, we assumed in making the figure that $T_{1/2} = (3.2 \pm 0.5) \times 10^{26}$ yr (case c above). The two other lifetimes considered above (case a and case b) result in pdfs with very similar shapes, but shifted relative to Fig. 5 to larger values of m , the lightest neutrino mass.

We can also extract from our analysis the allowed ranges

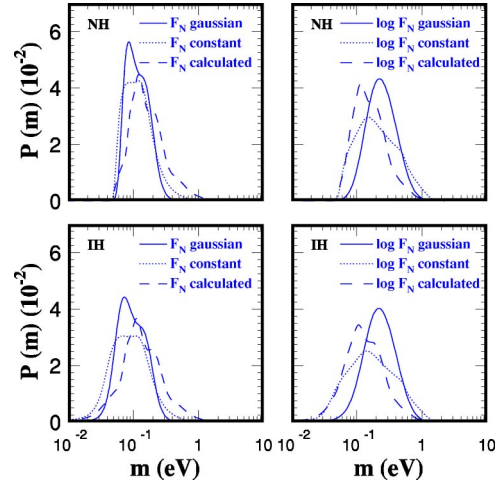


FIG. 5. Probability distributions of the lightest mass eigenstate m in a future generation neutrinoless double beta decay experiments. For illustration, we assumed a measured half life $T_{1/2}(^{76}\text{Ge}) = 3.2 \pm 0.5 \times 10^{26}$ yr. We obtained the distributions with the aid of Eq. (9) (see text for details). For the left hand panels, we assume that the probability distribution function for the nuclear factor F_N that appears in Eq. (1) satisfies a Gaussian distribution (full line), a constant distribution (dotted line), or the distribution of computed nuclear factor calculations by different theoretical groups (dashed line). The right hand panels correspond to assuming that $\log F_N$ follows those distributions. The upper panels correspond to a normal neutrino mass hierarchy [Eq. (7)], while the lower panels correspond to an inverted hierarchy [Eq. (5)].

of the total mass in neutrinos at a given C.L. Table IV presents the allowed ranges for the total mass in neutrinos for different assumptions regarding the pdfs of the nuclear factor F_N and for the three values of the half-life for neutrinoless double beta decay assumed above (cases a–c).

Figure 6 shows the cumulative probabilities for the total mass in neutrinos M ($M = m_1 + m_2 + m_3$). The results are illustrated for different assumptions regarding the pdf of F_N and for both normal and inverted neutrino mass hierarchies. In constructing Fig. 6, we assumed that $T_{1/2} = (3.2 \pm 0.5) \times 10^{26}$ yr (case c above). For the shorter half-lives corresponding to cases a and b above, the cumulative probabilities have very similar shapes, but are shifted to larger values of the total neutrino mass.

TABLE IV. Allowed ranges of the total mass in neutrinos for different assumed measurements of the half-life of ^{76}Ge to neutrinoless double beta decay. We consider different probability distributions of the nuclear factor: Gaussian, constant, or the actual distribution of 20 different theoretical calculations, using either a linear (lin) or a logarithmic (log) scale for F_N . In general, the results are different for normal and for inverted neutrino mass hierarchies. The results for the normal hierarchy are written in parentheses.

F_N pdf	M (eV) at 90 % C.L.	M (eV) at 95 % C.L.	M (eV) at 99 % C.L.	M (eV) at 99.73 % C.L.
$T_{1/2} = (3.2 \pm 0.2) \times 10^{25}$ yr				
actual, lin	[0.63,4.70] ([0.65,4.73])	[0.56,5.96] ([0.58,5.99])	[0.48,8.58] ([0.50,8.60])	[0.46,9.56] ([0.48,9.58])
actual, log	[0.64,4.22] ([0.66,4.24])	[0.57,5.36] ([0.59,5.39])	[0.49,7.84] ([0.51,7.88])	[0.46,9.41] ([0.48,9.43])
Gaussian, lin	[0.62,2.07] ([0.63,2.09])	[0.59,2.35] ([0.60,2.36])	[0.54,3.09] ([0.55,3.10])	[0.51,3.93] ([0.52,3.94])
Gaussian, log	[0.96,5.17] ([0.98,5.17])	[0.84,6.04] ([0.85,6.04])	[0.64,7.98] ([0.66,7.98])	[0.53,9.24] ([0.55,9.24])
constant, lin	[0.55,2.81] ([0.57,2.83])	[0.52,3.61] ([0.54,3.63])	[0.48,5.90] ([0.50,5.92])	[0.46,7.88] ([0.48,7.90])
constant, log	[0.63,6.45] ([0.65,6.48])	[0.57,7.74] ([0.59,7.75])	[0.51,9.45] ([0.53,9.45])	[0.48,9.83] ([0.50,9.83])
$T_{1/2} = (1.0 \pm 0.1) \times 10^{26}$ yr				
actual, lin	[0.34,2.76] ([0.37,2.81])	[0.30,3.63] ([0.33,3.69])	[0.26,6.23] ([0.29,6.30])	[0.24,8.34] ([0.28,8.40])
actual, log	[0.34,2.46] ([0.38,2.52])	[0.30,3.24] ([0.34,3.29])	[0.26,6.20] ([0.30,6.29])	[0.24,8.06] ([0.28,8.11])
Gaussian, lin	[0.34,1.16] ([0.36,1.18])	[0.32,1.32] ([0.34,1.34])	[0.29,1.73] ([0.31,1.76])	[0.28,2.21] ([0.30,2.24])
Gaussian, log	[0.53,2.93] ([0.56,2.94])	[0.46,3.43] ([0.49,3.45])	[0.34,4.65] ([0.38,4.66])	[0.28,5.70] ([0.32,5.71])
constant, lin	[0.29,1.56] ([0.33,1.61])	[0.28,2.01] ([0.31,2.07])	[0.26,3.35] ([0.29,3.43])	[0.24,4.66] ([0.28,4.73])
constant, log	[0.34,3.80] ([0.37,3.86])	[0.31,4.68] ([0.34,4.73])	[0.27,6.33] ([0.30,6.36])	[0.25,7.46] ([0.29,7.47])
$T_{1/2} = (3.2 \pm 0.5) \times 10^{26}$ yr				
actual, lin	[0.14,1.45] ([0.22,1.60])	[0.12,1.95] ([0.19,2.13])	[0.09,3.63] ([0.17,3.93])	[0.08,5.68] ([0.16,6.06])
actual, log	[0.15,1.29] ([0.22,1.42])	[0.12,1.72] ([0.20,1.85])	[0.09,3.31] ([0.17,3.56])	[0.08,4.51] ([0.16,4.67])
Gaussian, lin	[0.16,0.63] ([0.21,0.67])	[0.15,0.72] ([0.20,0.76])	[0.12,0.95] ([0.18,1.00])	[0.09,1.20] ([0.17,1.27])
Gaussian, log	[0.27,1.62] ([0.32,1.66])	[0.22,2.61] ([0.28,1.94])	[0.13,2.61] ([0.22,2.64])	[0.10,3.21] ([0.19,3.25])
constant, lin	[0.13,0.79] ([0.19,0.91])	[0.11,1.02] ([0.18,1.16])	[0.09,1.74] ([0.17,1.93])	[0.08,2.46] ([0.16,2.67])
constant, log	[0.16,2.03] ([0.22,2.16])	[0.14,2.53] ([0.20,2.65])	[0.10,3.51] ([0.18,3.60])	[0.09,4.18] ([0.17,4.26])

V. NORMAL OR INVERTED MASS HIERARCHY? NEUTRINOLESS DOUBLE BETA DECAY DETECTED, BUT NEUTRINO MASS NOT MEASURED

We make two assumptions in this section.

- (i) Next-generation experiments [16] will observe neutrinoless double beta decay.
- (ii) Next-generation ordinary beta decay experiments [35] will not detect the neutrino mass scale.

These assumptions are the opposite of what we postulated in Sec. III.

In this section, we answer the following question. Given the detection of neutrinoless double beta decay and the non-detection of a neutrino mass in normal beta decay, can we determine if the neutrino mass hierarchy is normal or inverted?

In order to answer this question, we computed the χ^2 distribution as a function of the different neutrino variables, including neutrino oscillation data where available for $\Delta m_{\odot}^2$, Δm_{atm}^2 , $\theta_{\odot}$, θ_{13} , the lightest mass m , the Majorana phases ϕ_1 and ϕ_2 , and the neutrinoless mass $|\langle m_{ee}^{\nu} \rangle|$. For an inverted (normal) neutrino mass hierarchy, we imposed Eq. (5) [Eq. (7)]. We then marginalized over all variables except m and $|\langle m_{ee}^{\nu} \rangle|$.

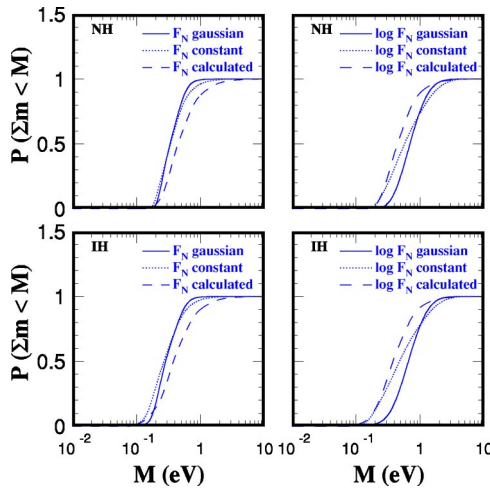


FIG. 6. The cumulative probability that the total mass in neutrinos is less than M . For illustration, we assumed a measured neutrinoless double beta decay half-life $T_{1/2}(^{76}\text{Ge}) = 3.2 \pm 0.5 \times 10^{26}$ yr. We calculated the cumulative probability by integrating Eq. (9) (see text for details). The organization of the panels and the notation are the same as for Fig. 5.

Figure 3 shows the allowed regions in the $|\langle m_{ee}^\nu \rangle|$ - m plane at 90 % C.L. for the inverted and normal hierarchy (full regions labeled IH and NH).

Just as we did in Sec. IV, we consider three cases of positive neutrinoless double beta detection with ^{76}Ge half-life: (a) $T_{1/2} = (3.2 \pm 0.2) \times 10^{25}$ yr, (b) $T_{1/2} = (1. \pm 0.1) \times 10^{26}$ yr, and (c) $T_{1/2} = (3.2 \pm 0.5) \times 10^{26}$ yr.

Normal and inverted neutrino mass hierarchies cannot be distinguished solely by a positive signal in a neutrinoless double beta decay next-generation experiment. This is illustrated by the dashed lines in Fig. 3 corresponding to cases a–c. All other things being equal, a relatively large value for $|\langle m_{ee}^\nu \rangle|$ favors an inverted hierarchy. However, for any experimentally accessible value of $|\langle m_{ee}^\nu \rangle|$ that is inferred from neutrinoless double beta decay, one can always postulate a sufficient large value of the lowest neutrino mass, m , that would account for the measured decay rate with mass-degenerate neutrinos.

In order to distinguish between a normal and an inverted neutrino mass hierarchy, we must somehow know that the lowest mass eigenstate m is very small (less than 0.01 eV). If we had a private communication showing that the lowest neutrino mass were zero, then we could distinguish between a normal and an inverted mass hierarchy. We find from detailed calculations that all three of the hypothetically successful measurements of a double beta decay lifetime [cases a, b, and c above] would, if $m = 0$, exclude a normal hierarchy independent of the pdf of the nuclear factor F_N .

VI. SUMMARY AND DISCUSSION

Next-generation neutrinoless double beta decay experiments offer the promise of a fundamental discovery, namely, that neutrinos are their own antiparticles. No other feasible experimental technique could establish this profound result. If a single experiment conclusively detects zero neutrino double beta decay, then weak interaction theory will be both profoundly simplified and greatly clarified.

Even if neutrinoless double beta decay is not observed in next-generation experiments, we may still be able to conclusively determine the particle and antiparticle nature of neutrinos. If an ordinary beta decay experiment detects a neutrino mass near 1 eV, then we will be able to conclude in this case that neutrinos are Dirac not Majorana particles.

In all other cases, the situation will be much less favorable, as can be seen readily from the summary given in Table I. If ordinary beta decay reveals a neutrino mass scale of less than 0.3 eV, then we will not be able to conclude that neutrinos are Dirac particles from the nonobservation of neutrinoless double beta decay in currently envisioned experiments. The particle and antiparticle nature of neutrinos will remain ambiguous.

The observation of neutrinoless double beta decay will determine a large allowed range of the total mass in the form of neutrinos, a range that permits an uncertainty in the total mass of between one and two orders of magnitude. This range translates into a total cosmic neutrino mass density (cf. [50]) $\Omega_\nu = 0.009 - 0.20$, $\Omega_\nu = 0.005 - 0.17$, or $\Omega_\nu = 0.0016$

– 0.12 at 3σ for the three assumed lifetimes listed in Table I and discussed in Sec. IV.

Finally, we note that we will not be able to decide whether the neutrino mass hierarchy is normal or inverted even if neutrinoless double beta decay is detected. In order to decide this important question, information from other types of experiments, such as long baseline oscillation studies, will be necessary.

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APPENDIX A: UPPER AND LOWER BOUNDS CONNECTED WITH NEUTRINOLESS DOUBLE BETA DECAY

In this appendix, we derive an upper bound, Sec. A 1, and a lower bound, Sec. A 2, on $|\langle m_{ee}^\nu \rangle|$. We assume that neutrinoless double beta decay is not observed in next generation experiments and that the neutrino mass hierarchy is inverted. In Sec. A 3, we obtain approximate results for the number of experiments that are required to show that neutrinos are Dirac particles using the inequalities derived in Secs. A 1 and A 2.

1. An upper bound on $|\langle m_{ee}^\nu \rangle|$

If a neutrinoless double beta decay experiment does not detect any events above the expected background, then the half-life satisfies

$$T_{1/2} \geq \frac{\Delta t \log 2}{-\log \alpha} N_X \epsilon, \quad (\text{A1})$$

where Δt is the period of data taking, N_X is the total number of active nuclei X , and ϵ is the efficiency of event capture after cuts to reduce background. The quantity $\alpha = 1 - \%$ C.L./100 is a given significance level. For definiteness, we will use the expectations for the Majorana experiment [5] to determine a reference sensitivity s to $T_{1/2} F_N$ [see Eq. (1)] in next generation neutrinoless double beta experiments [16]. The Majorana collaboration [5] is planning to use a 500 kg Ge (86 % ^{76}Ge) detector, $\Delta t = 5$ yr, and $\epsilon = 60\%$. With these values of the parameters, Eq. (A1) becomes

$$T_{1/2}(\text{Ge}) \geq \frac{7.13 \times 10^{27}}{-\log \alpha} \text{ yr}. \quad (\text{A2})$$

Different nuclear structure parameter F_N calculations of the transition in the case of ^{76}Ge , about 20, expand over a range (that we will consider as a 3σ range determination) of

$$F_N = (1.455 \pm 1.425) \times 10^{-13} \text{ yr}^{-1}. \quad (\text{A3})$$

The distribution of calculated values of F_N is shown in Fig. 1.

Inserting Eqs. (A2) and (A3) into Eq. (1), we find

$$|\langle m_{ee}^\nu \rangle| \leq 1.913 \times 10^{-2} \sqrt{\frac{-\log \alpha}{1.455 + 0.475n(\alpha)}} \text{ eV}, \quad (\text{A4})$$

where $n(\alpha)$ is the number of standard deviations at a given C.L., with an asymptotic expansion

$$n(\alpha) = \sqrt{\log\left(\frac{2}{\pi\alpha^2}\right) - \log\left[\log\left(\frac{2}{\pi\alpha^2}\right)\right]}. \quad (\text{A5})$$

For N neutrinoless double beta decay experiments N_{exp} with sensitivity to $|\langle m_{ee}^\nu \rangle|$ of s' , we have

$$|\langle m_{ee}^\nu \rangle| \leq 1.913 \times 10^{-2} \sqrt{\frac{s}{N_{exp}s'}} \sqrt{\frac{-\log \alpha}{1.455 + 0.475n(\alpha)}} \text{ eV}. \quad (\text{A6})$$

2. Inverted hierarchy: A lower bound

If the neutrino mass hierarchy is inverted, the neutrino mass element $|\langle m_{ee}^\nu \rangle|$ can be related to neutrino parameters determined in oscillation experiments [32–34,39,41,42] (see Fig. 3 for illustration) by the relation

$$|\langle m_{ee}^\nu \rangle| \geq \sqrt{\Delta m_{atm}^2 \cos 2\theta_\odot} \sim \left(\frac{-\log(1-\alpha)}{3 \times 10^{17}}\right)^{1/10}. \quad (\text{A7})$$

The fitting function with the exponent of 1/10 that appears in Eq. (A7) reproduces well the results obtained in the analysis of solar and atmospheric data [39]. The fitting function in Eq. (A7) deviates from the numerical results by less than 1 % in the range [20, 99.9] % C.L.

3. Approximate answer to the question posed in Sec. II

Equations (A6) and (A7) can be used to determine approximately the number of experiments N_{exp} with the expected sensitivity of the Majorana experiment that are required to show that neutrinos are Dirac particles if neutrinoless double beta decay is not observed and if the neutrino mass hierarchy is inverted. This question was answered by a brute-force method in Sec. II. By requiring that there be no intersection of the inequalities Eqs. (A6) and (A7), we calculate that the number of experiments required is $N_{exp} \geq 7, 12, 57$, and 645 at 90, 95, 99, and 99.73 % C.L., respectively. The approximate results obtained here are in good agreement with the more accurate results obtained in Sec. II and listed in Table II.

APPENDIX B: SENSITIVITY OF PROPOSED NEUTRINOLESS DOUBLE BETA DECAY EXPERIMENTS

Several next-generation neutrinoless double beta experiments have been proposed. Table V lists a representative

TABLE V. Proposed or suggested neutrinoless double beta decay experiments. The half-life sensitivities are estimated by the collaborations (with assumptions about backgrounds that have not yet been established experimentally) and scaled for five years of data taking (updated from Ref. [22]). The last column of the table presents the calculated sensitivities to the neutrinoless double beta decay mass matrix element, $|\langle m_{ee}^\nu \rangle|$. To compute results in the last column, we used the published distributions of calculated nuclear matrix elements [27] on a logarithmic scale. If less than three independent calculations of F_N have been published for a given nucleus, no estimate was computed for the sensitivity for that nucleus to the neutrino mass matrix element.

Experiment	Source	Sensitivity to $T_{1/2}$ (yr) at 90 % C.L.	Sensitivity to $ \langle m_{ee}^\nu \rangle $ (eV) at 90 % C.L.
CANDLES [6]	^{48}Ca	1×10^{26}	0.248
Majorana [5]	^{76}Ge	3×10^{27}	0.054
GEM [7]	^{76}Ge	7×10^{27}	0.034
GENIUS [8]	^{76}Ge	1×10^{28}	0.028
NEMO 3 [9]	^{100}Mo	4×10^{24}	0.646
MOON [10]	^{100}Mo	1×10^{27}	0.041
CAMEO [11]	^{116}Cd	1×10^{27}	0.057
COBRA [12]	^{130}Te	1×10^{24}	1.260
CUORICINO [13]	^{130}Te	1.5×10^{25}	0.336
CUORE [13]	^{130}Te	7×10^{26}	0.049
XMASS [14]	^{136}Xe	3×10^{26}	0.134
Xe [15]	^{136}Xe	5×10^{26}	0.104
EXO [16]	^{136}Xe	1×10^{28}	0.023
DCBA [17]	^{150}Nd	2×10^{25}	0.498
GSO [18,19]	^{160}Gd	2×10^{26}	

sample of different nuclei for which neutrinoless double beta decay experiments have been proposed (updated from Ref. [22]). The claimed sensitivity is shown in the third column of Table V, quantified by the half-life limit at 90 % C.L. in the case of negative searches. These limits have been evaluated using assumptions on background rates that have not yet been demonstrated experimentally and are scaled for five years of data taking. The comparison between different experiments should be made taking these considerations into account. The last column presents the sensitivity to the neutrinoless mass element, $|\langle m_{ee}^\nu \rangle|$. We used the distributions of calculated nuclear matrix element given in Ref. [27]. As far as we know, this has not been done in previous publications. In the literature, the translation from half-life to mass is usually made for a particular assumed nuclear matrix element factor. We think it may be useful to describe, as in Table V, the obtainable limits on the neutrinoless mass element by using the distribution of the calculated nuclear matrix elements that has been published for each nucleus.

Most nuclei have not yet been studied as widely as ^{76}Ge . We have determined a limit on the neutrinoless mass matrix element if there are three or more published nuclear matrix element calculations. We did not compute a limit for ^{160}Gd , for which we found only two published calculations [24,27].

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